



**Westwood  
Academy**

Together enabling the individual child

# Written Calculation Policy January 2016

# Westwood Academy Pencil and paper procedures

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of mathematical fluency. Mental arithmetic strategies for teaching mathematics will be taught systematically from Reception onwards and pupils will be given Daily opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at our school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions; use of jottings to aid a mental strategy; use of pencil and paper procedures

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose - pictures, mental calculation with or without jottings or structured recording.

## **Precision of mathematical Vocabulary:**

The correct terminology should be used when referring to the value of digits to support the children's understanding of place value.

Eg.  $68 + 47$  should be read 'sixty add forty' not 'six add four'

When using a columnar method. We refer to ones (1s) not units and identify strategies correctly, e.g. decomposition, recombine, partition etc.

**Aims :**

Children should be able to choose an efficient method, mental, written or calculator appropriate to the given task. By the end of year 6, children working at age related and above will have been taught and be secure with a compact standard method for each operation.

**General Progression :**

- Establish mental methods, based on a good understanding of place value
- Use of informal jottings to aid mental calculations
- Develop use of empty number line to help mental imagery and aid recording
- Use partitioning and recombining to aid informal methods
- Introduce expanded written methods
- Develop expanded methods into compact standard written form

Before carrying out a calculation, children will be encouraged to consider:

- Can I do it in my head? (using rounding, adjustment)
- The size of an approximate answer (estimation)
- Could I use jottings to keep track of the calculation?
- Do I need to use an expanded or compact written method?

**When are children ready for written calculations?****Addition and subtraction :**

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

**Multiplication and Division:**

- Do they know the 2,3,4,5 and 10 times tables?
- Do they know the result of multiplying by 1 and 0?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?
- These lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

**Problem solving, reasoning and mastery:**

Within every Maths lesson, children will have the opportunity to establish connections between the different areas of mathematics. This will see them provided with activities that secure their fluency of arithmetic strategies. Children will access a range of activities throughout a lesson to challenge their problem solving and reasoning skills. E.g.: 5-8 questions to rehearse calculation strategies, three or four questions that allow children to use and apply skills in a context and then a mastery opportunity to deepen their understanding. Children should be able to explain their thinking using the reasoning statements in the appendix to this policy. The NCETM mastery documents, NRich, Maths warrior website and Kangaroo Maths can be used to support mastery and mastery at a deeper level.

**Visualisation and Manipulatives:**

Where available, children will have access to clear boards and maths templates or maths writing frames, e.g. place value grids, number lines, to support their conceptual understanding of calculations and strategies which they will use to rehearse skills alongside teacher modelling. A range Manipulatives will be available in all lessons to develop this understanding of numbers and their value. E.g. Dienes/numicon/ counters/ coins.

## Stage 1

## Addition Stage 2

## Stage 3

### + = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$\begin{aligned} 2 &= 1 + 1 \\ 2 + 3 &= 4 + 1 \\ 3 &= 3 \\ 2 + 2 + 2 &= 4 + 2 \end{aligned}$$

Missing numbers need to be placed in all possible places.

$$\begin{array}{ll} 3 + 4 = 0 & 0 = 3 + 4 \\ 3 + 0 = 7 & 7 = 0 + 4 \\ 0 + 4 = 7 & 7 = 3 + 0 \\ 0 + 7 = 7 & 7 = 0 + 7 \end{array}$$

### Activities

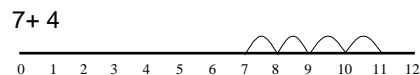
Children should have access to a wide range of counting equipment, everyday objects, as well as hoops, sorting trays, number tracks and numbered number lines.

### Teacher modelling

Drawing jumps on numbered number lines to support understanding of the mental method

### Children

To create their own jumps using rulers, fingers, pens, bodies etc.



### + = signs and missing numbers

Continue using a range of equations as in Stage 1 but with appropriate, larger numbers.

Extend to

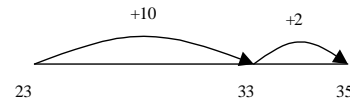
$$\begin{aligned} 14 + 5 &= 10 + 0 \\ \text{and} \\ 32 + 0 + 0 &= 100 \quad 35 = 1 + 0 + 5 \end{aligned}$$

### Partition into tens and ones and recombine

$$\begin{aligned} 12 + 23 &= 10 + 2 + 20 + 3 \\ &= 30 + 5 \\ &= 35 \end{aligned}$$

### Count on in tens and ones

$$\begin{aligned} 23 + 12 &= 23 + 10 + 2 \\ &= 33 + 2 \\ &= 35 \end{aligned}$$

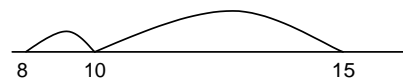


### Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10 e.g.

Children should be able to partition the 7 to relate adding the 2 and then the 5.

$$8 + 7 = 15$$

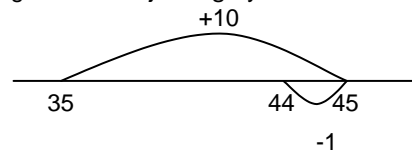


### Add 9 or 11 by adding 10 and adjusting by 1

e.g.

Add 9 by adding 10 and adjusting by 1

$$35 + 9 = 44$$

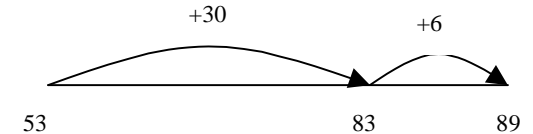


### + = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate, larger numbers.

### Partition into tens and ones

- Partition both numbers and recombine.
  - Count on by partitioning the second number only e.g.
- $$\begin{aligned} 36 + 53 &= 53 + 30 + 6 \\ &= 83 + 6 \\ &= 89 \end{aligned}$$



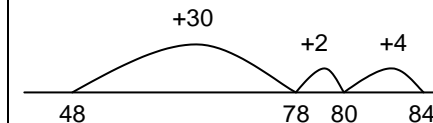
### Add a near multiple of 10 to a two-digit number

Secure mental methods by using a number line to model the method. Continue as in Stages 2 but with appropriate numbers

e.g.  $35 + 19$  is the same as  $35 + 20 - 1$ .

Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.

$$48 + 36 = 84$$



### pencil and paper procedures

$$83 + 42 = 125$$

either

or

1. Vertical expansion

$$\begin{array}{r} 83 \\ + 42 \\ \hline 5 \\ \hline 120 \\ 125 \end{array}$$

2. Horizontal expansion

$$\begin{aligned} 80 + 3 \\ + 40 + 2 \\ \hline 120 + 5 = 125 \end{aligned}$$

## Stage 4

## Addition Stage 5

## Stage 6

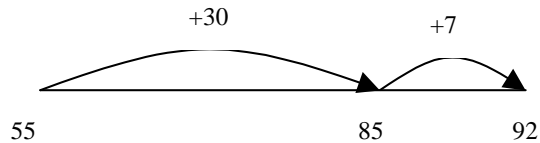
### + = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

### Partition into tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 55 + 37 &= 55 + 30 + 7 \\ &= 85 + 7 \\ &= 92 \end{aligned}$$



### Add the nearest multiple of 10, then adjust

Continue as in Stages 2 and 3 but with appropriate numbers e.g.  $63 + 29$  is the same as  $63 + 30 - 1$

### Pencil and paper procedures

$$367 + 185 = 431$$

either                      or

$$\begin{array}{r} 367 \\ +185 \\ \hline 12 \\ 140 \\ 400 \\ 552 \end{array} \qquad \text{or} \qquad \begin{array}{r} 300 + 60 + 7 \\ 100 + 80 + 5 \\ 400 + 140 + 12 = 552 \end{array}$$

leading to

$$\begin{array}{r} 367 \\ +185 \\ \hline 552 \\ 11 \end{array}$$

Extend to decimals in the context of money.

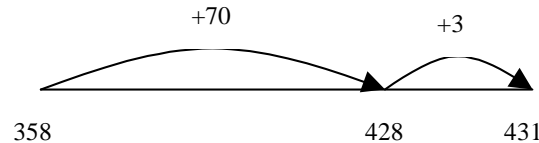
### + = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

### Partition into hundreds, tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 358 + 73 &= 358 + 70 + 3 \\ &= 428 + 3 \\ &= 431 \end{aligned}$$



### Add or subtract the nearest multiple of 10 or 100, then adjust

Continue as in Stages 2, 3 and 4 but with appropriate numbers e.g.  $458 + 79$  is the same as  $458 + 80 - 1$

### Pencil and paper procedures

Extend to numbers with at least four digits

$$3587 + 675 = 4262$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ 111 \end{array}$$

Revert to expanded methods if the children experience any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ +54.6 \\ \hline 127.4 \\ 11 \end{array}$$

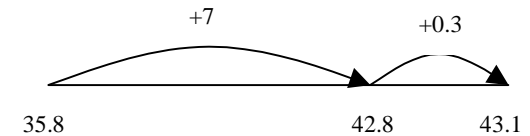
### + = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

### Partition into hundreds, tens, ones and decimal fractions and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 35.8 + 7.3 &= 35.8 + 7 + 0.3 \\ &= 42.8 + 0.3 \\ &= 43.1 \end{aligned}$$



### Add the nearest multiple of 10, 100 or 1000, then adjust

Continue as in Stages 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc

### Pencil and paper procedures

Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.

$$13.86 + 9.481 = 23.341$$

$$\begin{array}{r} 13.86 \\ + 9.481 \\ \hline 23.341 \\ 111 \end{array}$$

Revert to expanded methods if the children experience any difficulty.

## Stage 1

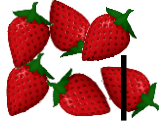
## Subtraction Stage 2

## Stage 3

### - = signs and missing numbers

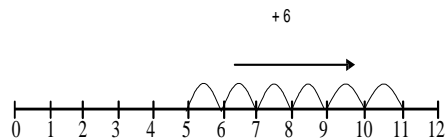
$$\begin{array}{ll} 7 - 3 = 0 & 0 = 7 - 3 \\ 7 - 0 = 7 & 4 = 0 - 3 \\ 0 - 3 = -3 & 4 = 7 - 0 \\ 0 - 7 = -7 & 4 = 0 - 7 \end{array}$$

- Understand subtraction as 'take away'



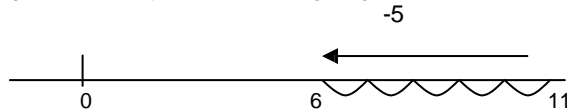
- Find a 'difference' by counting up;

I have saved 5p. The socks that I want to buy cost 11p. How much more do I need in order to buy the socks?



- Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number.

I have 11 toy cars. There are 5 cars too many to fit in the garage. How many cars fit in the garage?



Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences

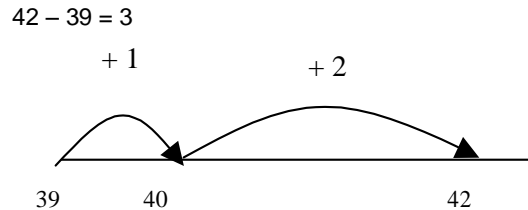
Recording by

- drawing jumps on prepared lines
- constructing own lines

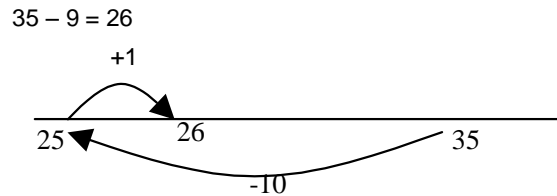
### - = signs and missing numbers

Continue using a range of equations as in Stages 1 but with appropriate numbers.  
Extend to  $14 + 5 = 20 - 0$

### Find a small difference by counting up

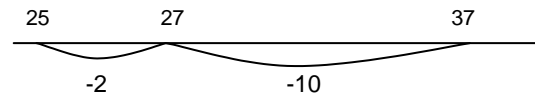


### Subtract 9 or 11. Begin to add/subtract 19 or 21

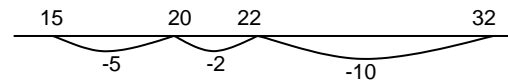


### Use known number facts and place value to subtract (partition second number only)

$$\begin{aligned} 37 - 12 &= 37 - 10 - 2 \\ &= 27 - 2 \\ &= 25 \end{aligned}$$



Bridge through 10 where necessary  
 $32 - 17$



### - = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

### Find a small difference by counting up

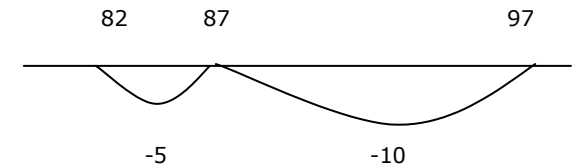
Continue as in Stages 2 but with appropriate numbers e.g.  
 $102 - 97 = 5$

### Subtract mentally a 'near multiple of 10' to or from a two-digit number

Continue as in Stages 2 but with appropriate numbers e.g.  $78 - 49$  is the same as  $78 - 50 + 1$

### Use known number facts and place value to subtract

Continue as in Stages 2 but with appropriate numbers e.g.  
 $97 - 15 = 72$

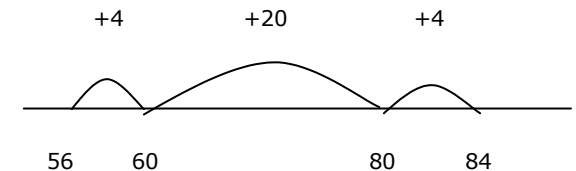


With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as  $57 - 12$ ,  $86 - 77$  or  $43 - 28$ .

### Pencil and paper procedures

Complementary addition

$$84 - 56 = 28$$







## Stage 1

Multiplication is related to doubling and counting groups of the same size.



Looking at columns  
 $2 + 2 + 2$   
 3 groups of 2

Looking at rows  
 $3 + 3$   
 2 groups of 3

### Counting using a variety of practical resources

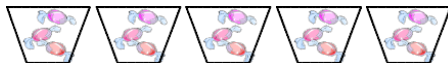
Counting in 2s e.g. counting socks, shoes, animal's legs...

Counting in 5s e.g. counting fingers, fingers in gloves, toes...

Counting in 10s e.g. fingers, toes...

Pictures / marks

There are 3 sweets in one bag.  
 How many sweets are there in 5 bags?



## Multiplication Stage 2

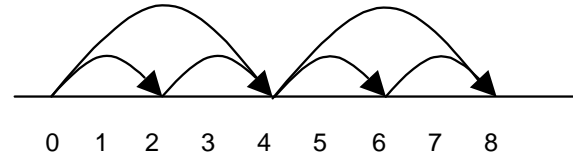
### x = signs and missing numbers

$$\begin{array}{ll} 7 \times 2 = 0 & 0 = 2 \times 7 \\ 7 \times 0 = 14 & 14 = 0 \times 7 \\ 0 \times 2 = 14 & 14 = 2 \times 0 \\ 0 \times 7 = 14 & 14 = 0 \times 7 \end{array}$$

### Arrays and repeated addition

$$\begin{array}{c} \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \end{array} \quad 4 \times 2 \text{ or } 4 + 4$$

$$2 \times 4 \text{ or } 2 + 2 + 2 + 2$$



### Doubling multiples of 5 up to 50

$$15 \times 2 = 30$$

### Partition

Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways:  $6 = 5 + 1$  so e.g. Double 6 is the same as double five add double one.



AND double 15

$$\begin{array}{r} 10 \quad + \quad 5 \\ \downarrow \quad \quad \downarrow \\ 20 \quad + \quad 10 = 30 \end{array}$$

OR

$$\begin{array}{r|l} X & 10 \quad 5 \\ \hline 2 & 20 \quad 10 \quad = 30 \end{array}$$

## Stage 3

### x = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers. Know by heart  $\times 2$ ,  $\times 3$ ,  $\times 4$ ,  $\times 5$  and  $\times 10$ . Multiply a single digit by 1, 10, 100

### Arrays and repeated addition

Continue to understand multiplication as repeated addition and continue to use arrays (as in Stages 2).

### Derive related facts

$$\begin{array}{l} 7 \times 5 = 35 \\ 5 \times 7 = 35 \\ 35 \div 7 = 5 \\ 35 \div 5 = 7 \end{array}$$

### Doubling multiples of 5 up to 50

$$35 \times 2 = 70$$

### Partition

$$\begin{array}{r|ll} X & 30 & 5 \\ \hline 2 & 60 & 10 & = 70 \end{array}$$

Use known facts and place value to carry out simple multiplications

Use the same method as above (partitioning), e.g.

$$32 \times 3 = 96$$

$$\begin{array}{r|l} X & 30 \quad 2 \\ \hline 3 & 90 \quad 6 & = 96 \end{array}$$

## Stage 4

## Multiplication Stage 5

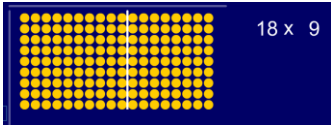
## Stage 6

### X = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers. Know by heart multiplication to  $12 \times 12$ .

### Partition

Continue to use arrays:



$$18 \times 9 = 162$$

$$18 \times 9 = (10 \times 9) + (8 \times 9) = 162$$

OR

Use the grid method of multiplication (as below)

### Pencil and paper procedures

Grid method

$23 \times 7$  is approximately  $20 \times 10 = 200$

x	20	3	
7	140	21	= 161

### x = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers.

### Partition

$$47 \times 6 = 282$$

$$47 \times 6 = (40 \times 6) + (7 \times 6) = 282$$

OR

Use the grid method of multiplication (as below)

### Pencil and paper procedures

Grid method

$72 \times 38$  is approximately  $70 \times 40 = 2800$

x	70	2	
30	2100	60	= 2160
8	560	16	= <u>576</u>
			2736

Extend to simple decimals with one decimal place.

### x = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers

### Partition

$$87 \times 6 = 522$$

$$87 \times 6 = (80 \times 6) + (7 \times 6) = 522$$

OR

Use expanded form leading to long multiplication.

### Pencil and paper procedures

Extended form

$$246$$

$$x \quad 7$$

$$1400$$

$$280$$

$$\underline{42}$$

$$\underline{1722}$$

### Leading to long multiplication

$$352$$

$$x \quad 27$$

$$2464$$

$$\underline{7040}$$

$$\underline{9504}$$

Extend to decimals with up to two decimal places.

$$x \quad 3$$

$$\underline{13.86}$$

$$1$$

## Stage 1

### Sharing

Requires secure counting skills  
 -see counting and understanding number strand  
 Develops importance of one-to-one correspondence  
 See appendix for additional information on x and ÷ and aspects of number

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

### Grouping

Sorting objects into 2s / 3s/ 4s etc  
 How many pairs of socks are there?



There are 12 crocus bulbs. Plant 3 in each pot. How many pots are there?  
 Jo has 12 Lego wheels. How many cars can she make?

## Division Stage 2

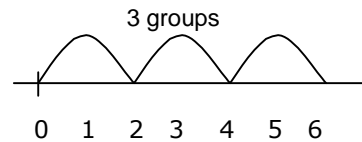
### ÷ = signs and missing numbers

$$\begin{array}{ll} 6 \div 2 = 0 & 0 = 6 \div 2 \\ 6 \div 0 = 3 & 3 = 6 \div 0 \\ 0 \div 2 = 3 & 3 = 0 \div 2 \\ 0 \div \nabla = 3 & 3 = 0 \div \nabla \end{array}$$

### Grouping

Link to counting and understanding number strand  
Count up to 100 objects by grouping them and counting in tens, fives or twos;...  
 Find one half, one quarter and three quarters of shapes and sets of objects  
 $6 \div 2$  can be modelled as:  
 There are 6 strawberries.  
 How many people can have 2 each? How many 2s make 6?

$6 \div 2$  can be modelled as:



In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game. How many teams are there?



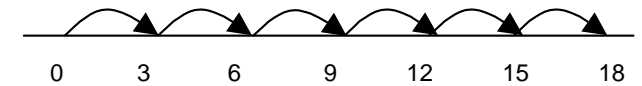
## Stage 3

### ÷ = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers.

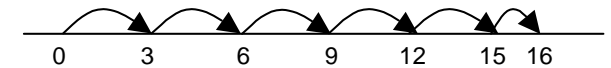
### Understand division as sharing and grouping

$18 \div 3$  can be modelled as:  
 Sharing – 18 shared between 3 (see Stages 1 diagram)  
 OR  
 Grouping - How many 3's make 18?



### Remainders

$16 \div 3 = 5 \text{ r}1$   
 Sharing - 16 shared between 3, how many left over?  
 Grouping – How many 3's make 16, how many left over?  
 e.g.



## Stage 4

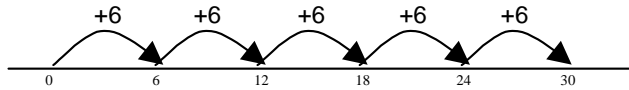
### ÷ = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers.

### Sharing and grouping

$30 \div 6$  can be modelled as:

grouping – groups of 6 placed on no. line and the number of groups counted e.g.



sharing – sharing among 6, the number given to each person

### Remainders

$$41 \div 4 = 10 \text{ r}1$$



$$41 = (10 \times 4) + 1$$

### Pencil and paper procedures

$72 \div 5$  lies between  $50 \div 5 = 10$  and  $100 \div 5 = 20$

\* Partition the dividend into multiples of the divisor:

e.g.  $72 = 50 + 22$

$$\begin{array}{r} 72 \\ \swarrow \searrow \\ 50 + 22 \\ \downarrow \downarrow \\ 10 + 4 \text{ r}2 \end{array}$$

$$\begin{array}{l} 50 \div 5 = 10 \\ 22 \div 5 = 4\text{r}2 \rightarrow 10 + 4\text{r}2 = 14 \text{ r}2 \\ \text{OR} \end{array}$$

$$\begin{array}{r} 72 \\ - \underline{50} \quad (10 \text{ groups}) \\ 22 \\ - \underline{20} \quad (4 \text{ groups}) \\ 2 \end{array}$$

Answer : 14 remainder 2

## Division Stage 5

### ÷ = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers.

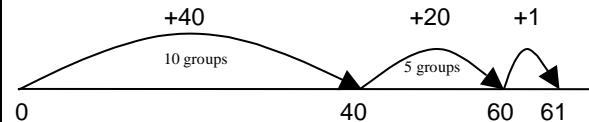
### Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

### Remainders

Quotients expressed as fractions or decimal fractions

$$61 \div 4 = 15 \frac{1}{4} \text{ or } 15.25$$



### Pencil and paper procedures

$196 \div 6$  is approximately just over  $180 \div 6 = 30$

Short division (HTU  $\div$  U)

$$196 \div 6$$

$$\begin{array}{r} \underline{32\text{r}.4} \\ 6 \overline{) 196} \end{array}$$

## Stage 6

### ÷ = signs and missing numbers

Continue using a range of equations as in Stages 2 but with appropriate numbers.

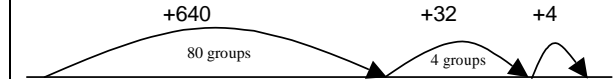
### Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

### Remainders

Quotients expressed as fractions or decimal fractions

$$676 \div 8 = 84.5$$



### Pencil and paper procedures

$434 \div 14$  is nearly  $450 \div 15 = 30$

Long division (HTU  $\div$  TU)

$$434 \div 14$$

$$\begin{array}{r} \underline{31} \\ 14 \overline{) 434} \\ \underline{-42} \phantom{0} \\ 14 \\ \underline{-14} \\ 0 \end{array}$$

Try out
14
$\times 3$
$\underline{42}$
1

## Sentence Structures To Develop Reasoning

I know...so I also know...	I found the pattern...	I know... so that means that ...	It follows that...
I found the pattern... so...	I think... because...	I thought... but now...	I agree with... because...
I... therefore...	This is similar to... so...	I considered... because... so now I will...	I disagree with... because...
I found...	I chose... because...	Perhaps... I could try...	...is important because...
When... it reminded me of...	We could test that by...	So...	It can't be... because...
I thought... but now I think...	because...	It could be... because...	First I did... then I tried...

I agree with...	I disagree with...	I decided...	I knew I needed to... and this helped me...
I think that...and...	I think that... but...	I noticed...	I decided to change...because...
I know... so I can work out...	I understand why you think... but have you considered... In that case...	I discovered... so...	It would work if...
if... then...	conclusion	...contradicts...	